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# Mass hierarchy and the spectrum of scalars

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**Abstract:**

We use the natural  $SU(3) \times U(1)$  global symmetry of the gauge-fermion interaction sector of the standard model to discuss the fermion mass hierarchy problem. The  $SU(3)$  sextet and triplet Higgs are introduced. The Yukawa sector is partially symmetric. The smaller the symmetry of a Yukawa term, the smaller its coupling constant. The mass hierarchy is a combined effect of smaller coupling constants and smaller VEVs. There is a bunch of pseudo-goldstone bosons which obtains their masses mainly from the small explicit breaking terms in the Higgs potential.

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The recent reports from CDF and D0 further confirm the existence of a heavy top quark[1]. Up to the present every particle in the minimal standard model[2,3,4] has been found except for the Higgs particle[5]. However many problems are still unresolved for the minimal standard model (MSM). Prominent among them is why quarks and leptons have specific hierarchical masses and very small mixing. Another unresolved problem is the exact mechanism of the electroweak symmetry breakdown, which, because of the lack of evidence of a Higgs particle, is the most uncertain in the MSM.

To attack the above two problems by relating one to another, S. Weinberg has proposed a multi-Higgs doublet model[6]. Noting that the mass of the top is, within a factor of two, close to that of the weak gauge bosons, and all the other quarks and leptons are much lighter, Weinberg assumes that the top ( $t$ ) and the weak gauge bosons,  $W$  and  $Z$ , obtain their masses mainly from the same vacuum expectation value (VEV) of a Higgs multiplet. The specific multiplet he chooses is a  $SU(2)_L$  doublet, and a  $SU(3)$  triplet at the same time. The  $SU(3)$  here is a global symmetry of his model. The Yukawa coupling term in the Lagrangian which serves the top a mass is  $SU(3)$  symmetric before a spontaneously symmetry breakdown (SSB). Therefore its coupling constant can be at the order of 1. The  $b$  quark also obtains its mass from the same expectation value. However the Yukawa coupling term which serves its mass is originally  $SO(3)$  symmetric, where  $SO(3)$  is a subgroup of the complete global symmetry  $SU(3)$ . Its coupling constant is therefore smaller, because the coupling term is less symmetric. All the other quarks and leptons should get their masses from other sources which do not enjoy such large global symmetries, and are from different VEVs (which are smaller). Indeed it is more natural to assume that masses with different orders of magnitude come from different sources than to account for them by one arbitrarily adjustable Yukawa coupling constant, which in the minimal standard model (which has the simplest possible scalar spectrum) runs from  $10^{-6}$  for the electron to  $10^0$  for the top. He then discusses the properties of the pseudo-goldstone bosons from the spontaneously symmetry breakdown of  $SU(3)$ . In particular, he points out that there is not a  $Z - Z - PGB$  coupling where  $PGB$  represents a pseudo-goldstone boson. Therefore, for instance, the LEP experiment cannot put a mass limit on these light particles, no matter how light they are, although their masses are at the order of  $10^2$  GeV, according to Weinberg. Thus Weinberg explains, in some sense, why the top quark is much heavier than the other quarks and leptons and the beauty is the

next heaviest, and he predicts a copious Higgs spectrum, in particular, a bunch of *PGBs*.

In this note we will devote ourselves to a similar line of thinking. We will introduce two modifications to Weinberg's original model:

1. We take

$$\mathbf{G} = SU(3) \times U(1) \tag{1}$$

as the global symmetry of the main part of the Lagrangian. We think that this is more natural, because the gauge-fermion interaction sector of the Lagrangian enjoys this bigger global symmetry. The basis of this symmetry lies in the fact that there are three families of quarks and leptons. In addition we follow Weinberg to assume that members of one family can transform differently under the  $SU(3)$  transformation. In other words, the three left-handed doublets of quarks are in a  $SU(3)$  triplet, while the three right-handed up (or down) type quarks may be, as in the model we are presenting, in a  $SU(3)$  anti-triplet. Therefore the global symmetry  $SU(3) \times U(1)$ , which we are talking about here is completely different from a family global symmetry. For a family group, all members in one family are collectively one object of the group transformation. In the sense of its changing an object in one family to another, this global symmetry is a horizontal symmetry.

2. We have one triplet Higgs as well as one sextet Higgs. The sextet Higgs will develop a large VEV, while the triplet, a small VEV. The advantage of introducing the sextet Higgs is that its big VEV naturally contributes to the big mass of the top and the relatively big mass of the beauty[6]. Further more, it explains the smallness of the weak mixing between the third and other generations straight forwardly (see later). The triplet Higgs in this model is instead responsible for the masses of the c and s quarks. An approximate value of  $V_{cb}$  is then calculable.

The bigger global symmetry  $\mathbf{G}$  allows us to have more explicit symmetry breaking terms in the Yukawa sector. We can write, in addition to an  $SO(3)$  symmetric Yukawa coupling term, also  $SU(3)$  and  $SU(2) \times U(1)' \times Z_2$  symmetric coupling terms. We therefore will be able to reproduce a mass matrix which is close to the one proposed by Fritzsch[7]. Since the VEVs of both Higgs multiplets contribute to masses of the quarks with the same electric charges, our model will have flavor-changed neutral currents mediated by pseudo-goldstone bosons at the tree level. However, as discussed by many authors, it should not be very

difficult to meet the most crucial experimental limits on flavor-changed neutral currents, if care is taken in model building[10].

As we emphasized the sector in our model which involves gauge interactions is completely standard, therefore, we will not discuss it. We will concentrate ourselves on the Higgs potential sector and the Yukawa sector. After presenting the model and exploring some of its features, we will briefly discuss the discovery channels for pseudo-goldstone bosons.

First let us give the fermion and scalar contents of the model in **Table 1**, where  $i$  and  $j$  are  $SU(3)$  indices which run from 1 to 3. All fields, except the standard gauge fields, and their global and gauge quantum numbers are listed in this table. From **Table 1** we see that we do not introduce any new fermions except those in the standard model. We just group them into the representations of the global symmetry  $\mathbf{G}$ . We also see that all the fermion  $SU(2)_L$  doublets (including quarks and leptons) are grouped into  $SU(3)$  triplets, while the  $SU(2)_L$  singlets (right-handed fermions) are grouped into  $SU(3)$  anti-triplets. For definiteness, we write here the explicit expressions for the Higgs fields:

$$\eta_i = \begin{pmatrix} \eta_i^+ \\ \eta_i^0 \end{pmatrix}, \quad \eta^{\dagger i} = \begin{pmatrix} \eta^{-i} & \eta^{0*i} \end{pmatrix}. \quad (2)$$

$$\Phi^{ij} = \begin{pmatrix} \phi^{+ij} \\ \phi^{0ij} \end{pmatrix}, \quad \Phi_{ij}^\dagger = \begin{pmatrix} \phi_{ij}^- & \phi_{ij}^{0*} \end{pmatrix}; \quad (3)$$

Now comes the Higgs potential, which we assume is  $\mathbf{G}$  symmetric when small explicit global symmetry breaking terms are neglected. We separate the symmetric potential into three parts, two self-interaction parts and one cross interaction part:

$$V_1(\Phi) = \delta Tr(\Phi^\dagger \Phi) + \beta (Tr(\Phi^\dagger \Phi))^2 + \alpha Tr(\Phi^\dagger \Phi)^2, \quad (\alpha + \beta > 0, \alpha + 3\beta > 0) \quad (4)$$

$$V_2(\eta) = \mu \eta^\dagger \eta + \lambda (\eta^\dagger \eta)^2, \quad (\lambda > 0) \quad (5)$$

$$V_{12}(\Phi, \eta) = \alpha_1 \sum_{ijk} \eta^{\dagger i} \Phi^{jk} \Phi_{ij}^\dagger \eta_k + \alpha_2 \sum_{ijk} \eta^{\dagger i} \eta_k \Phi_{ij}^\dagger \Phi^{jk} + \alpha_3 \eta^\dagger \eta Tr(\Phi^\dagger \Phi). \quad (6)$$

The positive definite conditions for  $V_1$  and  $V_2$  respectively are in parentheses. The conditions for the whole potential to be positive definite are more involved with the magnitudes of  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$ , compared with that of  $\alpha$ ,  $\beta$  and  $\lambda$ . We do not go into a detailed discussion of them here because they are not essential to the main subject of this note.

If both  $\delta$  and  $\mu$  in Eqs(4, 5) are negative, then both  $\Phi$  and  $\eta$  may develop VEVs. This is more so if  $\alpha_3 < 0$ . Let us suppose that one of the diagonal elements of the sextet Higgs develops VEV[9]. We call this component the 3-3 component,

$$\sqrt{2}\langle\Phi_{33}^0\rangle = v \sim 230GeV. \quad (7)$$

The effect of this VEV is to break the electroweak gauge symmetry  $SU(2)_L \times U(1)_Y$  down to  $U(1)_{em}$ , and to break the global symmetry  $\mathbf{G}$  down to  $SU(2) \times U(1)'$ . There are two possible patterns for  $\eta$  to develop a VEV:

$$\sqrt{2}\langle\eta_1^0\rangle = v', \quad (8)$$

or<sup>2</sup>

$$\sqrt{2}\langle\eta_3^0\rangle = v', \quad (9)$$

where  $v'$  could be complex<sup>3</sup> and we assume that the magnitude of  $v'$  is smaller than  $v$  (e.g. 2.5 times smaller). When  $\alpha_1 + \alpha_2 > 0$ , the first possibility is more favorable. This VEV pattern breaks the global symmetry further down to  $U(1)$ .  $U(1)_{em}$  is untouched by this VEV because this component in Eq(8) has the same  $SU(2)_L \times U(1)_Y$  property as the component in Eq(7).

Now we are ready to discuss the Yukawa sector. The Yukawa sector is not completely symmetric under the global transformations. Different Yukawa terms have different symmetry properties. The only term that is completely symmetric is

$$\mathbf{L}_0^Y = \Sigma_{ij} G_0 \bar{\psi}_L^i i\tau_2 \Phi_{ij}^\dagger U_R^j + h.c. \quad (10)$$

which after SSB will serve the top quark a large mass, if the magnitude of  $G_0$  is close to 1. A good reason for such a large Yukawa coupling is that this term is completely global symmetric.

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<sup>2</sup> $\langle\eta_2^0\rangle = v'$  is equivalent to  $\langle\eta_1^0\rangle = v'$ . It is just a matter of exchanging the definition of the first and second families.

<sup>3</sup>The condition to decide the phase of  $v'$  should be discussed elsewhere.

The following terms are symmetric under different subgroups of the global symmetry  $\mathbf{G}$

$$\begin{aligned}
\mathbf{L}_1^Y &= \Sigma_{ijk} G_1 \bar{\psi}_L^i \eta^{\dagger j} U_R^k \quad \varepsilon_{ijk} + h.c. \\
\mathbf{L}_2^Y &= \Sigma_{\alpha\beta\gamma\lambda=2,3} G_2 \bar{\psi}_L^\alpha \tau_{\alpha\gamma} \Phi^{\gamma\lambda} \tau_{\lambda\beta} D_R^\beta + h.c. \\
\mathbf{L}_3^Y &= \Sigma_{ijk} G_3 \bar{\psi}_L^i \eta_j D_R^k \quad \varepsilon_{ijk} + h.c.
\end{aligned} \tag{11}$$

where the  $G_1$  term is  $SU(3)$  symmetric, the  $G_2$  term is  $SU(2) \times U(1)' \times Z_2$  symmetric, and the  $G_3$  term,  $SO(3)$  symmetric. Therefore, according to the principle of naturalness, these coupling constants are sequentially smaller. The ratios of magnitudes of these couplings are about

$$G_0 : G_1 \sim 5, \quad G_1 : G_2 \sim 7, \quad G_2 : G_3 \sim 2.5. \tag{12}$$

Note that these ratios are all of the order of  $10^0$ . The following question can be answered here. In Eq(7) we assume that the component 3-3 of the sextet develops VEV. Here we assume that  $SU(2) \times U(1)' \times Z_2$  symmetry of the  $G_2$  term is on the 2 and 3 bases. Are these two assumptions consistent? In other words, in the summation of the  $G_2$  term, the indices  $\alpha$  etc. run over two values instead of three, in order to give the smaller symmetry. Why must these two values include 3 as one of them? Actually this is not an artificial choice. On the contrary, this is the only consistent choice. When the  $G_2$  term exists, there will be an induced term (and other terms) in the potential which is proportional to  $-|G_2|^2 \Sigma_{\alpha\beta} \Phi^{\alpha\beta} \Phi_{\alpha\beta}^\dagger$  where  $\alpha$  and  $\beta$  can only take 2 and 3. This induced term will make the component  $\Phi^{22}$  or  $\Phi^{33}$  more favorable to develop VEV.

The leptonic part of the Yukawa sector has only the  $G_1$  like and  $G_3$  like terms, if there are not right-handed neutrinos

$$\begin{aligned}
\mathbf{L}_{lep}^Y &= G_2^l \Sigma_{\alpha\beta\gamma\lambda} \bar{L}^\alpha \tau_{\alpha\gamma} \Phi_{\gamma\lambda} \tau_{\lambda\beta} l_R^\beta \\
&+ G_3^l \Sigma_{ijk} \bar{L}^i \eta_j l_R^k \quad \varepsilon_{ijk} + h.c..
\end{aligned} \tag{13}$$

We will not discuss the leptonic part further.

It is easy to read off the mass matrix for the up-type quarks, which is

$$M^U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_1 v'^* \\ 0 & -G_1^* v' & G_0 v \end{pmatrix}. \tag{14}$$

That for down-type quarks is

$$M^D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_3 v'^* \\ 0 & -G_3^* v' & G_2 v \end{pmatrix}. \quad (15)$$

The mechanism of producing the other matrix elements is still mysterious. In any case, these elements will be much smaller than those non-zero elements in the corresponding mass matrices. Therefore Eqs(14, 15) are very good approximations for the mass matrices. Note that in both matrices, the 3-3 elements obtain their contributions from the same big VEV, which naturally explains why both are the largest matrix elements in their respective mass matrices.

When diagonalizing the mass matrices we find that  $V_{cb}$  can be expressed as ( $V_{ub} = V_{td} = 0$  in this stage of approximation)

$$V_{cb} = x - x', \quad (16)$$

where  $x$  and  $x'$  are two complex numbers with their values related to the quark masses

$$|x| = \sqrt{m_c/m_t}, \quad |x'| = \sqrt{m_s/m_b}. \quad (17)$$

Note that the  $m_c$  obtained from  $M^U$  is at the order of  $|(G_2 v')^2/G_0 v|$ , which is much smaller than the smaller elements in  $M^U$ . Similarly,  $m_s$  is much smaller than the smaller elements in  $M^D$ . Therefore, small corrections to the zeroes in the mass matrices (14, 15) may cause the mass formulas for  $m_c$  and  $m_s$  to change an appreciable fraction, and consequently to change the formulas for  $x$  and  $x'$  appreciably, which may or may not improve the value of  $V_{cb}$  in Eqs(16, 17).

The masses of the psuedo-goldstone bosons in this model come mainly from the explicit symmetry breaking terms in the Higgs potential in this model. These asymmetric terms are supposed to break the continuous global symmetry completely, in order to avoid any goldstone particles that do not obtain masses after SSB. There are many possible terms, for example  $\mu' |\Sigma_i \eta_i|^2$ . Once the condition  $\mu' \ll \mu$  and  $\delta$  is satisfied, the basic picture discussed above will not be disturbed very much. The pseudo-goldstones obtain their masses also from the Yukawa interaction terms which break the corresponding global symmetry. However this contribution is much smaller than that from the explicit symmetry breaking terms in the Higgs potential. A limit for the masses of  $PGBs$  which mediate the  $b \rightarrow s$  transition

should in principle be able to be obtained from the data on  $B_s - \bar{B}_s$  mixing and  $b \rightarrow s + \gamma$ . The existence of flavor-changed neutral currents at the tree level are unavoidable because both up-type and down-type quarks obtain their masses from two different Higgs multiplets. However, a further discussion of this is out of the scope of this note.

Let us instead discuss briefly the interesting discovery channels of the  $PGBs$ . There are eight neutral  $PGBs$ <sup>4</sup>, when the global symmetry  $SU(3) \times U(1)$  spontaneously breaks down by the VEVs in Eqs(7, 8) to  $U(1)$ . As pointed out at the beginning of this note,  $PGBs$  cannot be discovered by  $Z \rightarrow Z^* + PGB$  or  $Z^* \rightarrow Z + PGB$  because there is no such coupling. The branching ratio of double  $PGB$  production on the  $Z$  peak is too tiny because it is a third order effect and the phase space for the final state is too small. The most interesting channel is top decay[1] and we have approximately

$$\frac{\Gamma(t \rightarrow c + PGB)}{\Gamma(t \rightarrow b + W^+)} \sim |x|^2 \left(\frac{m_t}{m_W}\right)^2, \quad (18)$$

if the relevant  $PGB$  is appreciably lighter than the top quark. So the decay into a pseudo-goldstone boson may be an appreciable channel of top decay. Such produced  $PGB$  will then decay into  $b + s$  or  $c + u$  etc. For  $PGBs$  which do not mediate flavor-changed neutral currents, their coupling constants to the light quarks are larger than that in the minimal standard model, because of the smallness of  $v'$ . Therefore they have a better chance to be found in hadron colliders.

In conclusion, we have presented here an alternative model which relates the scalar spectrum with the masses of quarks and leptons.  $SU(3) \times U(1)$  is used as the global horizontal symmetry. Left-handed fermions are in the triplet representations while right-handed fermions are in the anti-triplet representations. Both the Higgs triplet and sextet are introduced to provide VEVs with different values. The complete global symmetry is not secured in the Yukawa sector.  $SU(3) \times U(1)$  and  $SU(2) \times U(1)' \times Z_2$  symmetric Yukawa couplings are allowed for the sextet Higgs and  $SU(3)$  and  $SO(3)$  symmetric Yukawa couplings are allowed for the triplet Higgs. In this way we obtain second order mass matrices for both up- and down-type quarks.

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<sup>4</sup>There is no charged  $PGB$  because the global symmetry for the charged particles does not spontaneously break, for only the neutral components develop VEVs.



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**Table 1**

<i>object</i>	$SU(3)$	$SU(2)_L$	$U(1)^*$	$U(1)_Y$
$\psi_{Li}$	3	2	1	$\frac{1}{6}$
$L_i$	3	2	$\frac{2}{3}$	$-\frac{1}{2}$
$U_R^i$	$\bar{3}$	1	-1	$\frac{2}{3}$
$D_R^i$	$\bar{3}$	1	-1	$-\frac{1}{3}$
$l_R^i$	$\bar{3}$	1	$-\frac{4}{3}$	-1
$\Phi^{ij}$	$\bar{6}$	2	-2	$\frac{1}{2}$
$\eta_i$	3	2	1	$\frac{1}{2}$

\* The  $U(1)$  charge  $\xi = I - L/3$ , where I is the representation index of  $SU(3)$  and  $L$  is the lepton number.

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